1176-57-355 Yu-Chia Chen (yuchaz@outlook.com), Marina Meila* (mmp@stat.washington.edu) and Ioannis Kevrekidis. Finding shortest loop bases and the decomposition of the higher-order homology embeddings.

The study of the null space embedding of the graph Laplacian \mathcal{L}_0 has spurred new research and applications, such as spectral clustering algorithms with theoretical guarantees and estimators of the Stochastic Block Model. The null space of the k-th order Laplacian \mathcal{L}_k , known as the k-th homology vector space, encodes the non-trivial topology of a manifold or a network. In this work, we investigate the geometry of the k-th homology embedding and focus on cases reminiscent of spectral clustering. Namely, we analyze the connected sum of manifolds as a perturbation to the direct sum of their homology embeddings. We propose an algorithm to factorize the homology embedding into subspaces corresponding to a manifold's simplest topological components. The proposed framework is applied to the shortest homologous loop detection problem, a problem known to be NP-hard in general. Our spectral loop detection algorithm scales better than existing methods and is effective on diverse data such as point clouds and images. (Received January 25, 2022)