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Alex Cohen* (alexcoh@mit.edu). *A Sylvester-Gallai result for concurrent lines in the complex plane.*

The classical Sylvester-Gallai theorem says that for any finite set of points in \mathbb{R}^2 , not all lying on one line, there is some line passing through exactly two points of the set. These are called ordinary lines. This theorem fails over the complex numbers: there exist configurations of points in \mathbb{C}^2 so that every line passing through two of these points also passes through a third. There are only a few known examples of such sets, and it is an intriguing open problem to classify all of them. Very little is known in this direction, which leads us to ask: under what conditions must a set in \mathbb{C}^2 possess an ordinary line? In this work we show that if a set of points in \mathbb{C}^2 lies on m concurrent lines, and one of these lines contains more than $m - 2$ points, then the set admits an ordinary line. This bound is sharp and resolves a conjecture of Frank de Zeeuw. A key lemma shows that the boundary of a (suitably defined) convex hull in \mathbb{C}^2 has a tree structure. The proof of this lemma relies on a surprising application of Green's identity from calculus to the complex line. (Received January 24, 2022)