1176-51-318 **Jonah Gaster*** (gaster@uwm.edu) and **Tarik Aougab**. *Curves on the torus intersecting at most k times.*

A well-known counting problem in surface topology asks for the maximum number of simple closed curves that can fit on a closed surface of genus g so that every pair intersect at most k times – such a collection of curves is called a 'k-system'. Answers to this problem, even just for order of growth in k or g, are still unknown. The innocuous sounding special case $k = 1$ is a notoriously difficult problem, with notable recent progress from Przytycki and Greene.

Somewhat surprisingly, the complementary case $g = 1$ also remains mysterious. Ian Agol made the elegant observation that, on the torus, the size of a k-system is bounded by one more than the smallest prime larger than k, and via the Prime Number Theorem one can deduce that this quantity is asymptotic to k. We will discuss joint work with Tarik Aougab, in which we tighten the available upper bounds for $g = 1$, showing that a k-system on the torus has size at most $k + O(\sqrt{k}\log k)$. This matches the bound one would obtain via Agol's bound with the assumption of the Riemann hypothesis; by contrast our methods involve analysis of some combinatorial aspects of the associahedron and the hyperbolic geometry of the Farey complex. (Received January 25, 2022)