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**Fulton Gonzalez** (fulton.gonzalez@tufts.edu), **Jue Wang\*** (jue.wang@tufts.edu) and **Tomoyuki Kakehi** (kakehi@math.tsukuba.ac.jp). *Surjectivity of Convolution Operators on Noncompact Symmetric Spaces.*

In 1960, Ehrenpreis defined the *slow decrease* condition for functions on  $\mathbb{C}^n$ , and proved that for  $\mu \in \mathcal{E}'(\mathbb{R}^n)$ , the convolution operator  $C_\mu : T \mapsto T * \mu$  is surjective on  $\mathcal{E}(\mathbb{R}^n)$  as well as on  $\mathcal{D}'(\mathbb{R}^n)$ , if and only if the Fourier transform of  $\mu$  is of slow decrease. This generalizes the classical Malgrange-Ehrenpreis Theorem.

In this talk, we extend Ehrenpreis' result to noncompact symmetric spaces  $X = G/K$ . For each  $\mu \in \mathcal{E}'_K(X)$  (the subscript  $K$  means the  $K$ -invariance), let  $c_\mu : T \mapsto T * \mu$  be the right convolution operator on  $X$ . We establish the equivalence of the following:

- The spherical Fourier transform  $\tilde{\mu}$  is slowly decreasing.
- $c_\mu : \mathcal{E}_K(X) \rightarrow \mathcal{E}_K(X)$  is surjective;
- $c_\mu : \mathcal{D}'_K(X) \rightarrow \mathcal{D}'_K(X)$  is surjective;
- $c_\mu : \mathcal{E}(X) \rightarrow \mathcal{E}(X)$  is surjective;
- $c_\mu$  has a fundamental solution  $S \in \mathcal{D}'(X)$ , i.e.  $S * \mu = \delta_o$ .

As a corollary, any nonzero invariant differential operator always has a fundamental solution and is surjective on  $\mathcal{E}(X)$ , which are two classical results proved by Helgason in 1964 and 1973, respectively. (Received January 25, 2022)