Michael Allen (allenm3@math. oregonstate.edu), Renee Bell (rhbell@math.upenn.edu), Robert Lemke Oliver (robert.lemke_oliver@tufts.edu), Allechar Serrano Lopez* (serrano@math.harvard.edu) and Tian An Wong (tiananw@umich.edu). How do points on plane curves generate fields? Let me count the ways. Preliminary report.
In their program on diophantine stability, Mazur and Rubin suggest studying a curve $C$ over $\mathbb{Q}$ by understanding the field extensions of $\mathbb{Q}$ generated by a single point of $C(\overline{\mathbb{Q}})$; in particular, they ask to what extent the set of such field extensions determines the curve $C$. A natural question in arithmetic statistics along these lines concerns the size of this set: for a smooth projective curve $C / \mathbb{Q}$ how many field extensions of $\mathbb{Q}$ - of given degree and bounded discriminant - arise from adjoining a point of $C(\overline{\mathbb{Q}})$ ? Can we further count the number of such extensions with specified Galois group? Asymptotic lower bounds for these quantities have been found for elliptic curves by Lemke Oliver and Thorne, for hyperelliptic curves by Keyes, and for superelliptic curves by Beneish and Keyes. We discuss similar asymptotic lower bounds that hold for all smooth plane curves $C$, using tools such as geometry of numbers, Hilbert irreducibility, Newton polygons, and linear optimization. (Received January 24, 2022)

