1176-05-56 **Daniel McGinnis** and **Shira Zerbib*** (zerbib@iastate.edu), Department of Mathematics, Iowa State University, Ames, IA 50011. *A sparse colorful polytopal KKM theorem.* Preliminary report.

Recently Soberón proved a far-reaching generalization of the colorful KKM Theorem due to Gale: let $n \geq k$, and assume that a family of closed sets $(A_j^i \mid i \in [n], j \in [k])$ has the property that for every $I \in \binom{[n]}{n-k+1}$, the family $\left(\bigcup_{i\in I} A_1^i, \ldots, \bigcup_{i\in I} A_k^i\right)$ is a KKM cover of the $(k-1)$-dimensional simplex Δ^{k-1}; then there is an injection $\pi : [k] \to [n]$ so that $\bigcap_{i=1}^k A_i^{\pi(i)} \neq \emptyset$. We prove a polytopal generalization of this result, answering a question of Soberón in the same note. We also discuss applications of our theorem to fair division of multiple cakes, d-interval piercing, and a generalization of the colorful Carathéodory theorem. (Received January 11, 2022)