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*Transversals and colorings of simplicial spheres.*

Motivated from the surrounding property of a point set in  $\mathbb{R}^d$  introduced by Holmsen, Pach and Tverberg, we consider the transversal number and chromatic number of a simplicial sphere. As an attempt to give a lower bound for the maximum transversal ratio of simplicial  $d$ -spheres, we provide two infinite constructions. The first construction gives infinitely many  $(d + 1)$ -dimensional simplicial polytopes with the transversal ratio exactly  $\frac{2}{d+2}$  for every  $d \geq 2$ . In the case of  $d = 2$ , this meets the previously well-known upper bound  $1/2$  tightly. The second gives infinitely many simplicial 3-spheres with the transversal ratio greater than  $1/2$ . This was unexpected from what was previously known about the surrounding property. Moreover, we show that, for  $d \geq 3$ , the facet hypergraph  $\mathcal{F}(\mathbf{P})$  of a  $(d + 1)$ -dimensional simplicial polytope  $\mathbf{P}$  has the chromatic number  $\chi(\mathcal{F}(\mathbf{P})) \in O(n^{\frac{\lceil d/2 \rceil - 1}{d}})$ , where  $n$  is the number of vertices of  $\mathbf{P}$ . This slightly improves the upper bound previously obtained by Heise, Panagiotou, Pikhurko, and Taraz. (Received January 11, 2022)