1172-30-282 **Gaven J Martin*** (`g.j.martin@massey.ac.nz`), Institute for Advanced Study, Massey, University, Auckland, Auckland 0632, New Zealand, and **Cong Yao**. *The Ahlfors-Hopf differential, higher regularity for minimisers of exponential mean distortion and Teichmüller theory.* Preliminary report.

We consider minimisers of the p-exponential conformal energy for homeomorphisms $f : D \to D$ of finite distortion $K(z, f)$ with given boundary data $f_0 : D \to D$ and $f|S = f_0|S$,

$$E_p(f) = \int_D \exp[pK(z, f)]\, dz.$$

Homeomorphic minimisers always exist if the boundary data is a homeomorphism of finite energy. The Euler-Lagrange equations show that inverses $h = f^{-1}$ of sufficiently regular minimisers have an associated holomorphic quadratic differential - the Ahlfors-Hopf differential,

$$\Phi = \exp(p\, K(z, h) h_w \overline{h_{\bar{w}}}.$$

We establish that if $h : \Omega \to \mathbb{C}$ has holomorphic Ahlfors-Hopf differential, then h is a diffeomorphism and that h is harmonic in a metric induced by its own (smooth) distortion. A key fact is an analogue of the higher regularity theory for solutions of distributional inner variational equations established in our earlier work.

This allows us to link two different approaches to Teichmüller theory, namely the classical theory of extremal quasiconformal maps and the harmonic mapping theory. As $p \to \infty$ we recover the unique extremal quasiconformal mapping which is not a diffeomorphism however. As $p \to 0$ we recover the harmonic diffeomorphism in a homotopy class and Shoen-Yau's results. (Received August 30, 2021)