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Guangming Jing*, gjing1@gsu.edu, and **Zhongshan Li, Wei Gao, Wei Fang, Yanling Shao**
and **Yubin Gao**. *Convex polytopes and minimum ranks of nonnegative sign patterns.*

A sign pattern (matrix) \mathcal{A} is a matrix whose entries are from the set $\{+, -, 0\}$. A nonnegative sign pattern is a matrix whose entries are from the set $\{0, +\}$. The qualitative class of \mathcal{A} , denoted $Q(\mathcal{A})$, is defined as $Q(\mathcal{A}) = \{B \in M_n(\mathbb{R}) \mid \text{sgn}(B) = \mathcal{A}\}$. The minimum rank (resp., rational minimum rank) of a sign pattern matrix \mathcal{A} is the minimum of the ranks of the matrices (resp., rational matrices) whose entries have signs equal to the corresponding entries of \mathcal{A} . We show that for every nonnegative sign pattern of minimum rank at most 4, the minimum rank and the rational minimum rank are the same by using a correspondence between sign patterns with minimum rank r and point-hyperplane configurations in \mathbb{R}^{r-1} and Steinitz's theorem on the rational realizability of 3-polytopes,. We also show that every k -polytope corresponds to a nonnegative sign pattern with minimum rank $k+1$ that has a $(k+1) \times (k+1)$ triangular submatrix with all diagonal entries positive. Some bounds on the entries of the integer matrices achieving the minimum ranks of nonnegative sign patterns with minimum rank 3 or 4 are established. (Received January 27, 2019)