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Michael A Forbes* (miforbes@illinois.edu). *Explicit Dimension Reduction for Varieties, and the Polynomial Identity Testing Problem.*

We consider the task of mapping a large ambient space \mathbb{C}^n to small space \mathbb{C}^m so that a given variety X inside \mathbb{C}^n of dimension $\sim m$ has its relevant properties preserved under this map. In particular, a random linear map from $\mathbb{C}^n \rightarrow \mathbb{C}^m$ often suffices. Our challenge is to deterministically and efficiently construct such a linear map for interesting varieties.

(Mullmuley 2012) observed that for all “explicit” varieties this task is equivalent to the problem of developing deterministic algorithms for the polynomial identity testing (PIT) problem: given an algebraic circuit, decide whether this circuit computes the zero polynomial. I shall define the PIT problem, explain the connection to the above challenge, and outline why developing such deterministic algorithms is difficult. I will then define the ring of invariants for simultaneous conjugation of an r -tuple of $(n \times n)$ -matrices and instantiate Mullmuley’s result in this case, showing how one can express dimension reduction for this ring in the language of PIT. I will then describe how a deterministic PIT algorithm of (Forbes and Shpilka 2013) can give a near-solution to explicit dimension reduction for this ring.

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