edge-colorings of complete hypergraphs into regular colorings.
Let $\binom{X}{h}$ be the collection of all $h$-subsets of an $n$-set $X \supseteq Y$. Given a coloring (partition) of a set $S \subseteq\binom{X}{h}$, we are interested in finding conditions under which this coloring is extendible to a coloring of $\binom{X}{h}$ so that the number of times each element of $X$ appears in each color class (all sets of the same color) is the same number $r$. The case $S=\varnothing, r=1$ was studied by Sylvester in the 18th century, and remained open until the 1970s. The case $h=2, r=1$ is extensively studied in the literature and is closely related to completing partial symmetric Latin squares.

For $S=\binom{Y}{h}$, we settle the cases $h=4,|X| \geq 4.847323|Y|$, and $h=5,|X| \geq 6.285214|Y|$ completely. Moreover, we make partial progress toward solving the case where $S=\binom{X}{h} \backslash\binom{Y}{h}$. These results can be seen as extensions of the famous Baranyai's theorem, and make progress toward settling a 40 -year-old problem posed by Cameron. (Received February 13, 2018)

