Yan Cao, Department of Mathematics and Statistics, Georgia State University, Atlatna, GA 30303, and Guantao Chen* (gchen@gsu.edu), Department of Mathematics and Statistics, Georgia State University, Atlatna, GA 30303. Vizing's Average Degree Conjecture on Edge Chromatic Critical Graphs. Preliminary report.
Let $G$ be a simple graph. Denote by $\Delta(G), \delta(G)$, and $\chi^{\prime}(G)$ the maximum degree, minimum degree and the chromatic index of $G$. A graph $G$ is edge- $\Delta$-critical if $\chi^{\prime}(G)=\Delta+1$ and $\chi^{\prime}(H) \leq \Delta$ for any proper subgraph of $H$ of $G$. Let $\bar{d}(G)$ denote the average degree of $G$, Vizing in 1968 conjectured that the $\bar{d}(G) \geq \Delta-1+\frac{3}{n}$ if $G$ is an edge- $\Delta$-critical graph of order $n$. We show that if $G$ is an edge- $\Delta$-critical graph with $\Delta \geq 16$, then $\bar{d}(G) \geq \frac{3}{4} \Delta-8$. Moreover, we show that there exist two functions $D$ and $d$ such that for any positive real number $\epsilon \in(0,1)$, if $G$ is an edge- $\Delta$-critical graph with $\Delta \geq \operatorname{Delta}(\epsilon)$ and $\delta(G) \geq d(\epsilon)$, then $\bar{d}(G) \geq(1-\epsilon) \Delta$. We will give two specific functions satisfies the statement above. By using this theorem, we also show that an edge- $\Delta$-critical graph $G$ has $\bar{d}(G) \geq \Delta-o(\Delta)$ if $\delta(G) \geq(\log \Delta)^{\frac{3}{4}}$. (Received February 11, 2018)

