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Victor M. Bogdan and **Andrew E Bogdan*** (andrewbogdan@gmail.com). *Theory of Dirac Integral Spaces, derivation of ket and bra spaces as Hilbert spaces, relations to the theory of Lebesgue integral spaces.*

Let C be the space of complex numbers. A system $(X, F(C), M(C), L(C), \int, L_0(C))$ will be called a **Dirac Integral Space** if the following conditions are satisfied. (1) X is any abstract set. (2) $F = \{f|f : X \mapsto C\}$. (3) The subset $M \subset F$ has the property $(f_n \in M, f_n(x) \rightarrow f(x) \forall x \in X) \Rightarrow f \in M,$ and is closed under composition with continuous functions $u : C^m \mapsto C$. i.e $(f_j \in M, f(x) = u(f_1(x), \dots, f_m(x)) \forall x \in X) \Rightarrow f \in M$. Since the map $u(z) = |z|$ is continuous, we have $|f| \in M \forall f \in M$ where $|f|(x) = |f(x)| \forall x \in X$. (4) The set $L \subset M$ is linear and it is **solid** in M , that is, if $g \in M$ and $f \in L$ and $|g| \leq |f|$, then $g \in L$. (5) The functional $\int : L \mapsto C$ is linear and $\int f \geq 0$, if $f = |f|$, and for every series with terms $f_n \in L$ such that $\sum_n \int |f_n| < \infty$ and $\sum_n |f_n(x)| < \infty \forall x \in X$ and $f(x) = \sum_n f_n(x) \forall x \in X$, we have $f \in L$ and $\int f = \sum_n \int f_n$. (6) The set $L_0 = \{f \in L : \int |f| = 0\}$ is solid in the space F of all functions.

Using the above axioms, we will develop the theory of such spaces and prove that P.A.M. Dirac's ket space is a Hilbert space. (Received June 21, 2017)