

1131-11-187

Douglas Ulmer*, Department of Mathematics, University of Arizona, 617 N. Santa Rita Ave., Tucson, AZ 85721-0089. *Algebraic approaches to the Brauer-Siegel ratio for abelian varieties over function fields*. Preliminary report.

Let K be the function field of a curve \mathcal{C} over a finite field \mathbf{F}_q , and let A be an abelian variety over K . Analogy with the classical Brauer-Siegel theorem led Hindry to consider the ratio

$$BS(A) := \frac{\log(|III(A)|\text{Reg}(A))}{\log H(A)}$$

where $|III(A)|$ is the order of the Tate-Shafarevich group of A , $\text{Reg}(A)$ is the Néron-Tate regulator of A , and $H(A)$ is the exponential differential height of A .

If A_n , $n \geq 1$ is a sequence of abelian varieties of fixed dimension over K with $H(A_n) \rightarrow \infty$, the classical analogy would suggest that

$$\lim_{n \rightarrow \infty} BS(A_n) = 1.$$

Hindry, Pacheco, and Griffon have used analytic techniques to give several example of families $\{A_n\}$ for which the limit above exists and is equal to 1. (They also gave evidence for the conjecture that a limit of zero is possible.)

I will explain an algebraic technique for estimating $|III(A)|\text{Reg}(A)$ which recovers the limit above in all cases considered by Hindry, Pacheco, and Griffon. One novelty in our approach is that we use properties of the function

$$m \mapsto |III(A/\mathbf{F}_{q^m}(\mathcal{C}))|$$

to estimate $|III(A)|\text{Reg}(A)$ over the original ground field. (Received July 14, 2017)