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Karim Lounici*, Georgia Inst. of Technology, School of Mathematics, 686 Cherry St Ne,
Atlanta, GA 30332. *Concentration bounds and asymptotic distribution for the empirical spectral
projectors of sample covariance operators.*

Let X, X_1, \dots, X_n be i.i.d. Gaussian random variables in a separable Hilbert space \mathbb{H} with zero mean and covariance operator $\Sigma = \mathbb{E}(X \otimes X)$, and let $\hat{\Sigma} := n^{-1} \sum_{j=1}^n (X_j \otimes X_j)$ be the sample (empirical) covariance operator based on (X_1, \dots, X_n) . Denote by P_r the spectral projector of Σ corresponding to its r -th eigenvalue μ_r and by \hat{P}_r the empirical counterpart of P_r . Our goal is to obtain tight bounds on

$$\sup_{x \in \mathbb{R}} \left| \mathbb{P} \left\{ \frac{\|\hat{P}_r - P_r\|_2^2 - \mathbb{E}\|\hat{P}_r - P_r\|_2^2}{\text{Var}^{1/2}(\|\hat{P}_r - P_r\|_2^2)} \leq x \right\} - \Phi(x) \right|,$$

where $\|\cdot\|_2$ denotes the Hilbert–Schmidt norm and Φ is the standard normal distribution function. Such accuracy of normal approximation of the distribution of squared Hilbert–Schmidt error is characterized in terms of so called effective rank of Σ defined as $\mathbf{r}(\Sigma) = \frac{\text{tr}(\Sigma)}{\|\Sigma\|_\infty}$, where $\text{tr}(\Sigma)$ is the trace of Σ and $\|\Sigma\|_\infty$ is its operator norm, as well as another parameter characterizing the size of $\text{Var}(\|\hat{P}_r - P_r\|_2^2)$.

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