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**Romanos Diogenes Malikiosis** (malikios@math.tu-berlin.de), **Sinai Robins\*** (sinai\_robins@brown.edu) and **Yichi Zhang** (yczhang@ntu.edu.sg). *Polyhedral Gauss sums, and polytopes with symmetry.*

We define certain natural finite sums of  $n$ 'th roots of unity that are associated to each convex integer polytope  $P$ , and which generalize the classical 1-dimensional Gauss sum  $G(n)$  defined over  $\mathbb{Z}/n\mathbb{Z}$ , to integer polytopes. We call these finite exponential sums which  $G_P(n)$ . It is therefore natural to ask: for which convex integer polytopes do we get closed forms for  $G_P(n)$ , analogous to the 1-dim'l Gauss case? We consider the finite Weyl group  $W$ , generated by the reflections with respect to the coordinate hyperplanes, as well as all permutations of the coordinates, and we let  $G$  be the group generated by  $W$  as well as all integer vector translations.

We prove that if  $P$  multi-tiles  $\mathbb{R}^d$  under the action of the group  $G$ , then we have the closed form  $G_P(n) = \text{vol}(P)G(n)$ .

Conversely, we also prove that if the closed form expression  $G_P(n) = \text{vol}(P)G(n)$  is true for  $n \in \{1, 2, 3, 4\}$ , and  $P$  is a lattice tetrahedron in  $\mathbb{R}^3$  of (minimal) volume  $1/6$ , then there is an element  $g \in G$  such that  $g(P)$  is the fundamental tetrahedron with vertices  $(0, 0, 0)$ ,  $(1, 0, 0)$ ,  $(1, 1, 0)$ , and  $(1, 1, 1)$ . We will mention lots of open questions that arise naturally. (Received January 12, 2016)