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Frank G. Garvan* (fgarvan@ufl.edu). *Dyson's Conjectures and Predictions in the Work of Ramanujan.*

Let $p(n)$ be the number of partitions of n . Ramanujan proved that $p(5n + 4)$ is always divisible by 5 and that $p(7n + 5)$ is always divisible by 7. In 1944 Dyson defined the rank of a partition as the largest part minus the number of parts. Dyson conjectured that the residue of the rank mod 5 divides the partitions of $5n + 4$ into 5 equal classes, and that the residue of the rank mod 7 divides the partitions of $7n + 5$ into 7 equal classes, thus giving combinatorial refinements of Ramanujan's partition congruences. Dyson's rank conjectures were proved by Atkin and Swinnerton-Dyer in 1953. Atkin and Swinnerton-Dyer's main theorem is buried in an identity in Ramanujan's Lost Notebook. We show how this identity of Ramanujan can be extended to all primes greater than 3. It is also related to Dyson's 1987 prediction that there should be a group-theoretical structure, analogous to the structure of Hecke's theory of modular forms, for Ramanujan's mock-theta functions. In 2000 Zwegers made the breakthrough of realizing Ramanujan's mock theta functions as the holomorphic part of harmonic Maass forms. We extend Bringmann and Ono's work on Dyson's challenge, and show how experimental mathematics has played a crucial role. (Received January 17, 2016)