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The  $W_v$ -path conjecture states that any two vertices of a simple polytope can be joined by a path that does not revisit any facet. This is equivalent to the well-known Hirsch Conjecture. Klee conjectured even more, namely that the  $W_v$ -conjecture is true for all general cell complexes. Klee proved that the  $W_v$ -conjecture is true for 3-polytope (3-connected plane graphs). The general  $W_v$ -path conjecture was verified for projective plane and torus by Barnette, and the Klein bottle by Pulapaka and Vince. Recently, however, Santos disproved the Hirsch conjecture.

For every surface  $\Sigma$ , define a function  $f(\Sigma)$  such that if for every graph polyhedrally embedded in  $\Sigma$  and for every pair of vertices  $x$  and  $y$  in  $V(G)$ ,  $\kappa_G(x, y) \geq f(\Sigma)$ , then there exists a  $W_v$ -path joining  $x$  and  $y$ . Let  $\chi(\Sigma)$  be the Euler characteristic of  $\Sigma$ . We show that  $f(\Sigma) = 3$  if  $\Sigma$  is the sphere, and for all other surfaces  $3 - \tau(\Sigma) \leq f(\Sigma) \leq 9 - 4\chi(\Sigma)$ , where  $\tau(\Sigma) = \chi(\Sigma)$  if  $\chi(\Sigma) < -1$  and 0 otherwise. Further, if  $x$  and  $y$  are not cofacial, we show that  $G$  has at least  $\kappa_G(x, y) + 4\chi(\Sigma) - 8$  internally disjoint  $W_v$ -paths joining  $x$  and  $y$ . The bound is sharp for the sphere. (Received January 16, 2016)