48824. Descent and peak polynomials.

A permutation $\pi=\pi_{1} \ldots \pi_{n}$ in the summetric group $\mathfrak{S}_{n}$ has descent set $\operatorname{Des} \pi=\left\{i \mid \pi_{i}>\pi_{i+1}\right\}$. Given a set $S$ of positive integers and $n>\max S$, the descent polynomial of $S$ is the cardinality $d(S ; n)=\#\left\{\pi \in \mathfrak{S}_{n} \mid \operatorname{Des} \pi=S\right\}$. It is easy to prove, using the Principle of Inclusion and Exclusion, that this is a polynomial in $n$. However, properties of this polynomail do not seem to have been studied much in the literature. The peak set of $\pi$ is Pea $\pi=\left\{i \mid \pi_{i-1}<\pi_{i}>\pi_{i+1}\right\}$. Recently Billey, Burdzy, and Sagan proved that $\#\left\{\pi \in \mathfrak{S}_{n} \mid\right.$ Pea $\left.\pi=S\right\}=p(S ; n) \cdot 2^{n-\# S-1}$ where $p(S ; n)$ is a polynomial in $n$ which they dubbed the peak polynomial of $S$. These polynomials have since received the attention of a number of researchers. In this talk we will compare and contrast these two polynomials talking about their degrees, coefficients when expanded in the basis of binomial coefficients, roots, and analogues in other Coxeter groups. (Received January 13, 2017)

