1127-05-247 John Shareshian* (shareshi@math.wustl.edu) and Michelle L Wachs. Chromatic symmetric functions and regular semisimple Hessenberg varieties.

A Hessenberg vector is a weakly increasing sequence $m = (m_1, \ldots, m_n)$ of integers satisfying $i \leq m_i \leq n$ for all i. The graph G_m has vertex set [n], and an edge ij for each $1 \leq i < j \leq m_i$. Given an $n \times n$ complex matrix s, the Hessenberg variety V(m, s) is the subvariety of the flag variety consisting of all full flags $0 < V_1 < \ldots < V_n = \mathbb{C}^n$ satisfying $sV_i \leq V_{m_i}$ for all i.

When the matrix s is regular semisimple, the cohomology of V(m, s) admits an interesting representation of the symmetric group S_n . Brosnan and Chow proved a conjecture of Shareshian and Wachs, thus showing that (after tensoring with the sign character) the Frobenius characteristic of the given representation is (a refinement of) Stanley's chromatic symmetric function for G_m . (Another proof was given by Guay-Paquet.) This opens the door for a geometric attack on a conjecture of Stanley and Stembridge, which says that the chromatic symmetric function of G_m is a nonnegative integer combination of elementary symmetric functions. (Received February 05, 2017)