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The stochastic solution to a cauchy problem for degenerate parabolic equations.

This paper studies the stochastic solution to a Cauchy problem for degenerate parabolic equations in unbounded domains arising from option pricing. We first prove that “martingality of the underlying price process” is equivalent to “uniqueness of local stochastic solutions,” which in particular yields a non-standard Feynman-Kac formula. The stochastic solution, which represents the price of a European option, is shown to be a classical solution to the Cauchy problem, as long as the diffusion coefficient is locally Hölder continuous with exponent $\delta \in (0, 1]$. This improves the standard condition $\delta \geq 1/2$. A comparison theorem is also derived, without the usual linear growth condition on the diffusion coefficient. When the stochastic solution is not smooth, it is characterized as the limit of an approximating smooth stochastic solutions. Also noteworthy is that this paper presents a new proof for an equivalent condition for martingality of a one-dimensional diffusion process in natural scale. (Received January 18, 2016)