

1120-37-174

Alessandro Arsie* (alessandro.arsie@utoledo.edu), The University of Toledo, Department of Mathematics and Statistics, 2801 W Bancroft St. MS 942, Toledo, OH 43606, and **Paolo Lorenzoni** (paolo.lorenzoni@unimib.it), University of Milano-Bicocca, Department of Mathematics and Applications, Via Roberto Cozzi, 55, 20125 Milano, Milano, Italy. *F-manifolds, multi-flat structures and Painlevé transcendents.*

In this talk, we introduce F -manifolds equipped with multiple flat connections (and multiple F -products), that are required to be compatible in a suitable sense. Multi-flat F -manifolds are the analogue for F -manifolds of Frobenius manifolds with multi-Hamiltonian structures. In the semisimple case we show that a necessary condition for the existence of such multiple flat connections can be expressed in terms of the integrability (in the sense of the Frobenius Theorem) of a distribution of vector fields. When the relevant distributions are integrable, coupling the invariants of the foliations they determine with Tsarev's conditions, we construct bi-flat F -manifolds in dimension 2 and 3, and tri-flat F -manifolds in dimensions 3 and 4. We also obtain a parametrization of three-dimensional bi-flat F in terms of a system of six first order ODEs that can be reduced to the full family of P_{VI} equation. In the second part, we extend our analysis to include non-semisimple regular bi-flat and in general multi-flat F -manifolds. We show that in dimension three, regular non-semisimple bi-flat F -manifolds are locally parameterized by solutions of the full P_{IV} and P_V equations, according to the Jordan normal form of a distinguished endomorphism. (Received February 21, 2016)