Po-Shen Loh* (ploh@cmu.edu), Department of Mathematical Sciences, Carnegie Mellon University, Pittsburgh, PA 15213, and Jie Ma, School of Mathematical Sciences, University of Science and Technology of China. Diameter critical graphs.
A graph is called diameter- $k$-critical if its diameter is $k$, and the removal of any edge strictly increases the diameter. We prove several results related to a conjecture often attributed to Murty and Simon, regarding the maximum number of edges that any diameter- $k$-critical graph can have. In particular, we disprove a longstanding conjecture of Caccetta and Haggkvist (that in every diameter-2-critical graph, the average edge-degree is at most the number of vertices), which promised to completely solve the extremal problem for diameter-2-critical graphs.

On the other hand, we prove that the same claim holds for all higher diameters, and is asymptotically tight, resolving the average edge-degree question in all cases except diameter- 2 . We also apply our techniques to prove several bounds for the original extremal question, including the correct asymptotic bound for diameter- $k$-critical graphs, and an upper bound of $(1 / 6+o(1)) n^{2}$ for the number of edges in a diameter-3-critical graph. (Received January 20, 2015)

