1126-60-201 **Jason Cantarella**, **Thomas Needham** and **Clayton Shonkwiler*** (clay@shonkwiler.org), Colorado State University, Department of Mathematics, Campus Delivery 1874, Fort Collins, CO 80523, and **Gavin Stewart**. *Concavity, a question of Sylvester, and how to generate random quadrilaterals.*

"Show that the chance of 4 points forming the apices of a reentrant quadrilateral is 1/4 if they be taken at random in an indefinite plane."

This was Mathematical Question #1491 from the April, 1864 issue of the Educational Times, posed by J. J. Sylvester. Cayley provided a solution, but many others including De Morgan's son proposed alternative fractions as the correct answer, and after some spirited debate Sylvester came to believe that his question, as stated, does not "admit of a determinate solution".

While Sylvester asked about four points chosen randomly in the plane, perhaps a better question would be: is there a reasonable way of generating random quadrilaterals and, if so, what is the probability that a random quadrilateral is reentrant (or, if you like, concave)? As a special case of a more general theory which was originally developed to study random knots and polymers, we can identify quadrilaterals with points on the Grassmann manifold $G_2(\mathbb{R}^4)$ of planes in 4-dimensional space and, taking advantage of the symmetries of this space, give a satisfying answer to this modified version of Sylvester's question. This also leads to a new duality operation on quadrilaterals, the implications of which remain unexplored. (Received January 13, 2017)