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Ruth E Davidson* (redavid2@illinois.edu), Augustine O'Keefe (augustine.okeefe@conncoll.edu) and Daniel Parry (dan.t.parry@gmail.com). A new shellability proof of an old identity of Dixon.

We give a new proof of an old identity of Alfred Cardew Dixon (1865-1936). The new proof uses tools from topological combinatorics. Dixon's identity is re-established by constructing a family of non-pure shellable simplicial complexes $\Delta(n)$, whose structure is a function of any positive integer n. The alternating sum of the numbers of faces of $\Delta(n)$ of each dimension is the left-hand side of the the identity, and we show that the alternating sum of the Betti numbers of the complex is equal to the right-hand side of the identity. In other words, Dixon's identity is re-established by using the Euler-Poincaré relation for $\Delta(n)$. The Betti numbers are calculated by showing that for any n, $\Delta(n)$ is shellable. Then, using the well-known fact that a (possibly non-pure) shellable simplicial is homotopy equivalent to a wedge of spheres, we count the number of faces of $\Delta(n)$ of each dimension that attach along their entire boundary-also known as homology facets-in the shelling order, thereby computing the Betti numbers of $\Delta(n)$. This is joint work with Augustine O'Keefe and Daniel Parry. (Received August 13, 2015)