1107-11-481 Patrick Joseph Dynes* (pdynes@clemson.edu), 110 Cherry Hill Avenue, Goose Creek, SC 29445, and Brian J McDonald (bmcdon11@u.rochester.edu) and Kimsy Tor (ktor.student@manhattan.edu). Gaussian Behavior of Generalized Zeckendorf Decompositions over Small Scales.
A beautiful theorem of Zeckendorf states that every positive integer can be written uniquely as a sum of non-consecutive Fibonacci numbers $\left\{F_{n}\right\}$, with initial terms $F_{1}=1, F_{2}=2$. We consider the distribution of the number of summands involved in such decompositions. Previous work proved that as $n \rightarrow \infty$ the distribution of the number of summands in the Zeckendorf decompositions of $m \in\left[F_{n}, F_{n+1}\right)$, appropriately normalized, converges to the standard normal.

We generalize these results to subintervals of $\left[F_{n}, F_{n+1}\right)$ as $n \rightarrow \infty$. Explicitly, fix an integer sequence $\alpha(n) \rightarrow \infty$. As $n \rightarrow \infty$, for almost all $m \in\left[F_{n}, F_{n+1}\right)$ the distribution of the number of summands in the Zeckendorf decompositions of integers in the subintervals $I_{m ; n}:=\left[m, m+F_{\alpha(n)}\right)$, appropriately normalized, converges to the standard normal. The proof follows by showing that, with probability tending to $1, m$ has at least one appropriately located large gap between indices in its decomposition. We then use a correspondence between this interval $I_{m ; n}$ and $\left[0, F_{\alpha(n)}\right)$ to obtain the result, since the summands are known to have Gaussian behavior in the latter interval. We also prove the same result for more general linear recurrences. (Received January 20, 2015)

