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Alexander I. Nazarov* (al.il.nazarov@gmail.com). *On the existence of extremal function in the Maz'ya–Sobolev inequality.*

Denote by $x = (y; z)$ a point in $R^n = R^m \times R^{n-m}$, $n \geq 3$, $2 \leq m \leq n-1$. By P we denote the subspace $\{x \in R^n : y = 0\}$.

Let G be a domain in R^n . We denote by $\dot{W}_p^1(G)$ the closure of $C_0^\infty(G)$ w.r.t. the norm $\|\nabla v\|_{p,G}$.

For $0 \leq s \leq \min\{1, \frac{n}{p}\}$ we put $p_s^* = \frac{np}{n-sp}$. We discuss the sharp constant in the Maz'ya–Sobolev inequality

$$\| |y|^{s-1} v \|_{p_s^*, G} \leq N \|\nabla v\|_{p, G}.$$

Note that the case $p < n$, $s = 1$ gives usual Sobolev inequality. For $m = 1$ our problem degenerates in a sense. For $m = n$ see the survey [1].

For $p < n$ and $0 < s < 1$ the sharp constant easily is not attained for any G provided $G \cap P \neq \emptyset$.

We consider more complicated case $G \cap P = \emptyset$, $\partial G \cap P \neq \emptyset$. First, we deal with G being a wedge $\mathcal{K} = K \times R^{n-m}$ (K is an open cone in R^m) or a perturbed wedge for all $1 < p < \infty$ and $0 \leq s < \min\{1, \frac{n}{p}\}$. Then we consider the case of a bounded domain for $p = 2$ and $0 < s < 1$.

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REFERENCES

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