

1089-20-179

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*Generating functions for real character degree sums of finite general linear and unitary groups.*

It is known that if  $\chi$  is any real-valued irreducible complex character of  $GL(n, q)$ , then  $\chi$  is the character of a real representation, that is, the Frobenius-Schur indicator of  $\chi$  is 1. It follows that the sum of the degrees of these real-valued characters of  $GL(n, q)$  is equal to the number of elements in the group which square to the identity, which we can count. On the other hand, we may use symmetric functions to obtain a generating function for the degree sum for real-valued characters. In the case that  $q$  is even, we use  $q$ -series identities to obtain a new proof that all real-valued characters of  $GL(n, q)$  have indicator 1. When  $q$  is odd, we instead apply this known result to obtain what seems to be a new  $q$ -series identity.

In the case of the finite unitary group  $U(n, q)$ , the Frobenius-Schur indicators of its characters in general are unknown. We compute a generating function for the sum of the real character degrees for this group, again using symmetric function theory, and also by applying the results for  $GL(n, q)$  and a change of variables  $q \mapsto -q$ . In the end, we obtain a generating function for the sum of the degrees of real-valued characters of  $U(n, q)$  which have indicator 1, and one for the sum of those with indicator  $-1$ . (Received February 13, 2013)