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Keith M Fox* (kmcauliffefox@gmail.com). *A Characterization of The Witnesses to The Non-Normality of \mathbb{N}^{ω_1}* . Preliminary report.

We expand on A.H. Stone's 1948 result that \mathbb{N}^{ω_1} is not normal, by characterizing the closed sets which witness the non-normality of \mathbb{N}^{ω_1} . We give necessary and sufficient conditions on closed sets $Z \subset \mathbb{N}^{\omega_1}$ for the existence and construction of a closed set $Z' \subset \mathbb{N}^{\omega_1}$ where Z' is disjoint from Z , and where Z and Z' witness the non-normality of \mathbb{N}^{ω_1} . We use the above to get a relationship between witnesses to the non-normality of \mathbb{N}^{ω_1} and discrete subsets of \mathbb{N}^{ω_1} , as well as give a pair of disjoint countable closed discrete subsets of \mathbb{N}^{ω_1} which can not be separated by open sets having disjoint closures. A.H. Stone gave two closed disjoint homeomorphic subsets of \mathbb{N}^{ω_1} failing to have an open separation. We expand on the property that his witnesses are homeomorphic by showing that for any closed set $Z \subset \mathbb{N}^{\omega_1}$ if Z has a non-Lindelöf boundary, then there exists a closed set $Z' \subset \mathbb{N}^{\omega_1}$ where Z and Z' are disjoint, homeomorphic, and fail to have an open separation. (Received July 30, 2011)