1073-14-2 **Allen Knutson*** (`allenk@math.cornell.edu`). *Modern developments in Schubert calculus.*

How many k-planes in n-space are there satisfying a list of (sufficiently generic) intersection conditions? This is a product calculation in the cohomology ring of a Grassmannian. One might say that (e.g.) the number of red lines in projective 3-space that touch four generic blue lines is *topologically* constrained to be 2.

This has many generalizations, e.g. replacing "k-planes V" by "isotropic k-planes" or "chains ($V_1 < V_2 < \ldots$) of subspaces"; replacing cohomology by e.g. K-theory or quantum cohomology; or any combination. The multiplication's coefficients no longer have such simple geometric interpretations, but in each case, *algebraic geometry* shows that they are nonnegative.

For many purposes (e.g. applications to control theory), one is only interested in whether some k-plane can be found at all. So one wants rules for these numbers that are not alternating sums but manifestly positive, and *algebraic combinatorics* enters the fray.

I'll focus on one rule (of many known) that Terry Tao, Chris Woodward, and I found useful for constraining sums of Hermitian matrices. This "puzzle" rule has a very direct connection to the algebraic geometry, and (by adding puzzle pieces) extensions to several of the generalizations. (Received August 01, 2011)