Given a $k$-uniform hypergraph (or a $k$-graph for short) $H$ and a positive integer $n$, the Turán number $e x_{k}(n, H)$ of $H$ is the maximum number of edges in a $k$-graph $\mathcal{F}$ on $n$ vertices that does not contain $H$ as a subhypergraph. The Turán problem for hypergraphs is difficult and $e x_{k}(n, H)$ is asymptotically determined only for very few graphs. Exact values are known only for a handful of $k$-graphs $H$, most of which are on a small number of vertices.

A $k$-uniform linear path $\mathcal{P}_{\ell}^{k}$ of length $\ell$ is a $k$-graph with hyperedges $F_{1}, \ldots, F_{\ell}$ such that $\left|F_{i} \cap F_{i+1}\right|=1$ for all $i$ and $F_{i} \cap F_{j}=\emptyset$ whenever $|i-j|>1$. Frankl determined $e x_{k}\left(n, \mathcal{P}_{\ell}^{k}\right)$ when $\ell=2$. Here, we determine $e x_{k}\left(n, \mathcal{P}_{\ell}^{k}\right)$ exactly for all fixed $\ell \geq 1, k \geq 4$, and sufficiently large $n$. We show that $e x_{k}\left(n, \mathcal{P}_{2 t+1}^{k}\right)=\binom{n-1}{k-1}+\binom{n-2}{k-1}+\ldots+\binom{n-t}{k-1}$ and $\operatorname{ex}\left(n, \mathcal{P}_{2 t+2}^{k}\right)=\binom{n-1}{k-1}+\binom{n-2}{k-1}+\ldots+\binom{n-t}{k-1}+\binom{n-t-2}{k-2}$. We also describe the unique extremal graphs and establish stability results on these bounds. Our main method is the delta-system method. (Received August 02, 2011)

