1073-05-136 **Richard Hammack*** (`rhammack@vcu.edu`), Virginia Commonwealth University, Dept. of Mathematics and Applied Mathematics, Box 842014, Richmond, VA 23284. *The factorial of a graph.*

In 1971 Lovász proved the following cancellation law for the direct product of graphs: If A, B and C are graphs, then $A \times C \cong B \times C$ implies $A \cong B$, provided C has an odd cycle. This gives exact conditions on C that govern whether cancellation holds or fails.

Left unresolved were the conditions on A (or B) that guarantee cancellation. We introduce a new construction, called the *factorial of a graph*, that settles this issue.

The factorial of a graph A is a certain graph $A!$ defined on the set Perm(A) of permutations of $V(A)$. We define the edges and indicate some parallels to the factorial operation on integers. In fact, the edge set $E(A!)$ is a group that acts naturally on Perm(A). We show how (for bipartite C) the orbits of this action are in one-to-one correspondence with the graphs B for which $A \times C \cong B \times C$; if the action is transitive, then cancellation holds.

In addition to solving the cancellation problem, the factorial raises many interesting questions. Given a finite group G, is there a graph A with $E(A!) \cong G$? Can we characterize those A for which $E(A!)$ is (say) abelian? Are such questions significant or mere curiousities? There are more unknowns than knowns. (Received July 30, 2011)