1073-05-101David Galvin* (dgalvin1@nd.edu), University of Notre Dame, and Do Trong Thanh,
University of Notre Dame. Graph Stirling numbers. Preliminary report.

For a graph G and an integer k, the graph Stirling number S(G, k) is the number of partitions of V(G) into k non-empty independent sets. Equivalently it is the number of proper colourings of G using exactly k colours, with two colourings identified if they differ only on the names of the colours. If G is the empty graph on n vertices then S(G, k) is just S(n, k), the Stirling number of the second kind.

Harper showed that the Stirling numbers are asymptotically normal: if X_n takes value k with probability proportional to S(n,k), then $(X_n - \mu_n)/\sigma_n$ tends in distribution to a standard normal (where μ_n and σ_n are the mean and standard deviation of X_n). We obtain an analog of this result for Stirling numbers of forests with not too many components. The first step is to show that the generating function of the sequence of Stirling numbers of a forest has all real roots. This is a consequence of a result of Brenti; we give a more direct proof that exhibits some nice patterns among the roots.

We also consider Stirling numbers of cycles. An involved argument of Brenti, Royle and Wagner established that the sequence of Stirling numbers of a cycle is log-concave; we give a very direct proof. (Joint work with Do Trong Thanh.) (Received July 28, 2011)