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Let k be any field, G be a finite group acting on the rational function field $k(x_g : g \in G)$ by $h \cdot x_g = x_{hg}$ for any $h, g \in G$. Define $k(G) = k(x_g : g \in G)^G$. Noether's problem asks whether $k(G)$ is rational (= purely transcendental) over k . It is known that, if $C(G)$ is rational over C , then $B_0(G) = 0$ where $B_0(G)$ is the unramified Brauer group of $C(G)$ over C . Bogomolov showed that, if G is a p -group of order p^5 , then $B_0(G) = 0$. This result was disproved by Moravec for $p = 3, 5, 7$ by computer computing. Without using computers, we will prove two results. Theorem 1. Let p be any odd prime number. Then there is a group G of order p^5 satisfying $B_0(G) \neq 0$. In particular, $C(G)$ is not rational over C . Theorem 2. Let G be a group of order 243 other than $G(243, i)$ where $56 \leq i \leq 60$. Let e be the exponent of G , and k be a field containing a primitive e -th root of unity. Then $k(G)$ is rational over k if and only if $B_0(G) = 0$, i.e. G is not isomorphic to $G(243, i)$ with $28 \leq i \leq 30$ (where $G(243, i)$ is the GAP code number for the i -th group of order 243). (Received October 04, 2011)