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In 1954 Roger Lyndon gave the earliest example of a finite algebra, denoted here by  $\mathbf{L}$ , with a nonfinitely based equational theory. Z. Szekely noticed 40 years later that  $\mathbf{L}$  is associated with a finite automaton and so belongs to the class of automatic algebras. Roughly speaking the equational complexity of an algebra  $\mathbf{A}$  is a function  $\beta(n)$  measuring how much of the equational theory of  $\mathbf{A}$  must be examined to determine whether an algebra of size bounded by  $n$  belongs to the variety generated by  $\mathbf{A}$ . The finite algebra membership problem for  $\mathbf{A}$  is the problem of deciding, given a finite algebra  $\mathbf{B}$ , whether  $\mathbf{B}$  belongs to the variety generated by  $\mathbf{A}$ . We show the following about Lyndon's algebra:

1. The equational complexity of  $\mathbf{L}$  lies between  $n - 4$  and  $2n + 1$ .
2. The finite algebra membership problem of  $\mathbf{L}$  is decidable in nondeterministic logarithmic space (and hence in polynomial time).
3. The variety generated by  $\mathbf{L}$  has subdirectly irreducible algebras of every cardinality bigger than 1 except 4.
4.  $\mathbf{L}$  is inherently nonfinitely based relative to the variety generated by all automatic algebras.

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