Michael T. Lacey (Lacey@math.gatech.edu), School of Mathematics, Georgia Institute of Technology, Atlanta, GA 30332, Istvan Prause (Istvan. Prause@helsinki.fi), P.O. Box 68 (Gustaf Hällströmin katu 2b), University of Helsinki, FI-00014 Helsinki, Finland, Eric T. Sawyer (sawyer@mcmaster.ca), Dept. of Mathematics \& Statistics, McMaster University, 1280 Main Street West, Hamilton, Ontario L8S 4K1, Canada, Xavier Tolsa (xtolsa@mat.uab.cat), Departament de Matematiques, Universitat Autonoma de Barcelona, 08193 Bellaterra Barcelona, Spain, and Ignacio Uriarte-Tuero* (ignacio@math.msu.edu), Department of Mathematics, Michigan State University, East Lansing, MI 48824. Two conjectures of Astala on distortion under planar quasiconformal mappings and related removability problems.
In his celebrated paper on area distortion under planar quasiconformal mappings (Acta 1994), Astala proved that if $E$ is a compact set of Hausdorff dimension $d$ and $f$ is $K$-quasiconformal, then $f E$ has Hausdorff dimension at most $d^{\prime}=\frac{2 K d}{2+(K-1) d}$, and that this result is sharp. He conjectured (Question 4.4) that if the Hausdorff measure $\mathcal{H}^{d}(E)=0$, then $\mathcal{H}^{d^{\prime}}(f E)=0$.

UT showed that Astala's conjecture is sharp in the class of all Hausdorff gauge functions (IMRN, 2008).
Lacey, Sawyer and UT jointly proved completely Astala's conjecture in all dimensions (Acta, 2010). The proof uses Astala's 1994 approach, geometric measure theory, and new weighted norm inequalities for Calderón-Zygmund singular integral operators which cannot be deduced from the classical Muckenhoupt $A_{p}$ theory.

These results are related to removability problems for various classes of quasiregular maps. I will mention sharp removability results for bounded $K$-quasiregular maps (i.e. the quasiconformal analogue of the classical Painleve problem) recently obtained jointly by Tolsa and UT.

I will further mention recent results related to another conjecture of Astala on Hausdorff dimension of quasicircles obtained jointly by Prause, Tolsa and UT. (Received August 03, 2010)

