Simon Brendle* (brendle@math.stanford.edu), Stanford University, 450 Serra Mall, Bldg 380, Stanford, CA 94305. Curvature, sphere theorems, and the Ricci flow.

In 1926, Hopf proved that any compact, simply connected Riemannian manifold with constant curvature 1 is isometric to the standard sphere. Motivated by this result, Hopf posed the question of whether a compact, simply connected manifold with suitably pinched curvature is topologically a sphere. This question has been studied by many authors over the past six decades, a milestone being the Topological Sphere Theorem proved by Berger and Klingenberg in 1960.

In this lecture, I will discuss the history of this problem, and describe the proof (joint with R. Schoen) of the Differentiable Sphere Theorem. This theorem classifies all manifolds with 1/4-pinched curvature up to diffeomorphism. The distinction between homeomorphism and diffeomorphism is significant in light of the exotic spheres constructed by Milnor; the proof uses the Ricci flow technique introduced by Hamilton. (Received March 13, 2010)