Alexandr Kostochka* (kostochk@math. uiuc.edu), 1409 W. Green St., Department of Mathematics, Urbana, IL 61801, Mohit Kumbhat (kumbhat2@uiuc.edu), 1409 W. Green St., Department of Mathematics, Urbana, IL 61801, and Tomasz Luczak (tomasz@amu.edu.pl), Faculty of Mathematics and Computer Science, ul. Umultowska 87, 61614 Poznan, Poland. Conflict-free colorings of uniform hypergraphs with few edges.
A coloring of the vertices of a hypergraph $\mathcal{H}$ is conflict-free if for each edge $e$ of $\mathcal{H}$, some color appears on exactly one vertex of $e$. The smallest number of colors required for such a coloring is called the conflict-free chromatic number, $\chi_{C F}(\mathcal{H})$, of $\mathcal{H}$. It turned out that conflict-free chromatic number has interesting applications and interesting behavior. Pach and Tardos studied this parameter for graphs and hypergraphs. Among other things, they proved that for each ( $2 r-1$ )-uniform hypergraph $\mathcal{H}$ with $m$ edges, $\chi_{C F}(\mathcal{H})$ is at most $C m^{1 / r} \log m$. They also asked whether the same result holds for $r$-uniform hypergraphs. We show that this is not true. Furthermore, we provide new lower and upper bounds on the minimum number of edges in an $r$-uniform simple hypergraph that is not conflict-free $k$-colorable. (Received August 02, 2010)

