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*Independent varieties of loops.*

We characterise joins of disjoint varieties  $\mathcal{V}_1, \mathcal{V}_2$  of loops. The problem amounts to characterising independent varieties of loops. We obtain:

**Theorem 1**  $\mathcal{V}_1$  and  $\mathcal{V}_2$  are independent if and only if there exist unary terms  $s(x)$  and  $s'(x)$ , satisfying

1.  $\mathcal{V}_1 \models s(x) = x, s'(x) = s'(y) = e,$

2.  $\mathcal{V}_2 \models s(x) = s(y) = e, s'(x) = x.$

For varieties of power-associative loops with inverse property we can do better:

**Theorem 2**  $\mathcal{V}_1$  and  $\mathcal{V}_2$  are independent if and only if there is an integer  $\ell$  such that  $\mathcal{V}_1 \models x^\ell = e$  and  $\mathcal{V}_2 \models x^{\ell-1} = e$ . Moreover, if both  $\mathcal{V}_1$  and  $\mathcal{V}_2$  are nontrivial, then  $\ell > 2$ .

**Theorem 3** The following are equivalent.

1.  $\mathcal{V}$  satisfies the identities  $x^{k(k-1)} = e$  and  $(xy)^{1-k}(zu)^k = x^{1-k}z^ky^{1-k}u^k$  for some  $k > 2$ .

2.  $\mathcal{V} = \mathcal{V}_1 \times \mathcal{V}_2$ , for nontrivial independent varieties  $\mathcal{V}_1$  and  $\mathcal{V}_2$ .

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