John Engbers* (jengbers@nd.edu), Department of Mathematics, 255 Hurley Hall, Notre Dame, IN 46556, and David Galvin. The typical structure of $H$-colorings of the Hamming cube.
The $d$-dimensional discrete hypercube $Q_{d}$ is the graph on $\{0,1\}^{d}$ with two strings adjacent if they differ on one coordinate. For a graph $H$ (possibly with loops), an $H$-coloring of $Q_{d}$ is a function from $\{0,1\}^{d}$ to $V(H)$ which preserves adjacency. With appropriate choices of $H, H$-colorings can encode independent sets and proper colorings of $Q_{d}$.

We are interested in the following question: In a uniformly chosen $H$-coloring of $Q_{d}$, what proportion of vertices of $Q_{d}$ get mapped to each vertex of $H$ ? We obtain a quite precise answer to this question. For example, we can say that in a uniformly chosen proper $2 k$-coloring of $Q_{d}$, asymptotically almost surely each color class has size very close to $2^{d} /(2 k)$, and in a uniformly chosen proper ( $2 k+1$ )-coloring, asymptotically almost surely there are $k$ color classes with size very close to $2^{d} /(2 k)$ and $k+1$ class with size very close to $2^{d} /(2(k+1))$. In both cases, each color class is contained almost exclusively in a single bipartition class of $Q_{d}$.

The results generalize to the discrete torus with fixed even side length. The approach is through entropy, and extends results obtained by Jeff Kahn (who had considered the case when $H$ is a doubly infinite path). (Received September 08, 2010)

