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Princeton, NJ 08540. *On representations of integers in thin subgroups of  $SL(2, Z)$ .*

The Affine Linear Sieve extends sieve methods to thin orbits of non-abelian group actions. The fundamental work of Bourgain-Gamburd-Sarnak showed in great generality that one may obtain an infinitude of  $R$ -almost primes (numbers with at most  $R$  prime factors), but without specifying  $R$ . Some explicit values of  $R$  were obtained in specific thin situations by Kontorovich and Kontorovich-Oh, but unconditionally were still in the teens (i.e.  $R = 14$ ). In this work, we obtain an infinitude of actual primes in such an orbit. We show that not only do infinitely many primes appear, but that almost every number appears, as long as it is not excluded by congruence conditions; moreover the number of exceptions has a power savings. The main ingredients are the circle method, estimates for certain bilinear forms, the spectral gap, and effective sector estimates for hyperbolic isometries. (Received September 15, 2009)