1048-22-63   **Konstanze Rietsch** and **Lauren Williams***, Department of Mathematics, Harvard University, 1 Oxford Street, Cambridge, MA 02138. *Discrete Morse theory for totally nonnegative flag varieties.*

In a seminal 1994 paper, Lusztig extended the theory of total positivity by introducing the totally non-negative part $(G/P)_{\geq 0}$ of an arbitrary (generalized, partial) flag variety G/P. He referred to this space as a "remarkable polyhedral subspace," and conjectured a decomposition into cells, which was subsequently proven by the first author. In this article we use discrete Morse theory to show that the cell decomposition of $(G/P)_{\geq 0}$ is polyhedral in the following sense: closures of cells are collapsible and hence contractible. This answers a question posed by Lusztig in 1996, and generalizes a later result of Lusztig's, that $(G/P)_{\geq 0}$ – the closure of the top-dimensional cell – is contractible. Furthermore, we show that the boundary of each cell – hence in particular the boundary of $(G/P)_{\geq 0}$ – is homotopy equivalent to a sphere. (Received January 20, 2009)