1048-16-348   **Stefan Forcey**, Tennessee State University, **Aaron Lauve*** (`lauve@math.tamu.edu`), Texas A&M University, and **Frank Sottile**, Texas A&M University. *Hopf structures on binary trees (variations on a theme).*

We discuss several algebraic structures that can be placed on the vertices of the multiplihedra $\{\mathcal{M}_n\}$, a family of polytopes defined by Stasheff in the study of higher categories and homotopy theory. The vertices may be indexed by certain "bi-leveled" binary trees, intermediate between ordered trees and ordinary (planar binary) trees. The structures we find stem from the relationship between these three types of trees.

The ordered trees (i.e., permutations) and ordinary trees index the vertices of two additional families of polytopes more familiar to combinatorists: the permutahedra $\{\mathcal{P}_n\}$ and the associahedra $\{\mathcal{A}_n\}$. They also arrange themselves into Hopf algebras (after work of Malvenuto–Reutenauer and Loday–Ronco, respectively). In this talk, we give $\mathcal{M}$ the structure of $\mathcal{P}$-module and $\mathcal{A}$-Hopf module algebra in a manner respecting the cellular maps $\mathcal{P}_n \to \mathcal{M}_n \to \mathcal{A}_n$. We also give a basis of coinvariants for a second $\mathcal{A}$-Hopf module structure on $\mathcal{M}$. (Received February 10, 2009)

1