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Branch-and-bound proposed in the 1960s is a classical method to find an integral point in a polyhedron, and more generally, to solve integer programming problems. On the theoretical side, however, it has been deemed hopelessly inefficient: it can take an exponential number of nodes to prove that a simple polyhedron does not contain a lattice point. As a result, theoretically efficient algorithms, such as Lenstra's and Kannan's algorithms, and the Lovasz-Scarf generalized basis reduction method rely on fairly involved techniques.

In this work we show that branch-and-bound is theoretically efficient, if we apply a simple transformation in advance to the constraint matrix which makes the columns short and near orthogonal. We prove that if the coefficients of the problem are drawn from a sufficiently large interval, then for almost all such instances the number of subproblems that need to be enumerated by branch-and-bound is at most one.

Besides providing an analysis of a classical algorithm, our result also extends a theorem of Furst and Kannan on the solvability of subset sum problems. (Received November 27, 2008)