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Gilles Pisier*, Texas A & M University and Université Paris VI. *Complex interpolation between Hilbert, Banach and operator spaces.*

Let $B(X, Y)$ denote the Banach space of bounded operators between two Banach spaces X, Y . We describe the complex interpolation spaces

$$(B(\ell_{p_0}), B(\ell_{p_1}))^\theta \text{ or } (B(L_{p_0}), B(L_{p_1}))^\theta$$

for any pair $1 \leq p_0, p_1 \leq \infty$ and $0 < \theta < 1$. In the same vein, given a locally compact Abelian group G , let $M(G)$ (resp. $PM(G)$) be the space of complex measures (resp. pseudo-measures) on G equipped with the usual norm $\|\mu\|_{M(G)} = |\mu|(G)$ (resp.

$$\|\mu\|_{PM(G)} = \sup\{|\hat{\mu}(\gamma)| \mid \gamma \in \widehat{G}\}.$$

We describe similarly the interpolation space $(M(G), PM(G))^\theta$. Various extensions and variants of this result will be given, e.g. to Schur multipliers on $B(\ell_2)$ and to operator spaces. Motivated by a question of Vincent Lafforgue, we study the Banach spaces X satisfying the following property: there is a function $\varepsilon \rightarrow \Delta_X(\varepsilon)$ tending to zero with $\varepsilon > 0$ such that every operator $T: L_2 \rightarrow L_2$ with $\|T\| \leq \varepsilon$ that is simultaneously contractive (i.e. of norm ≤ 1) on L_1 and on L_∞ must be of norm $\leq \Delta_X(\varepsilon)$ on $L_2(X)$. We show that $\Delta_X(\varepsilon) \in O(\varepsilon^\alpha)$ for some $\alpha > 0$ iff X is isomorphic to a quotient of a subspace of an ultraproduct of θ -Hilbertian spaces for some $\theta > 0$ where θ -Hilbertian is meant in a slightly more general sense than in our previous work. (Received May 27, 2008)