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The Kac model considered here models the collisions of  $N$  particles with three dimensional velocities by a random walk in which the steps correspond to binary collisions that conserve momentum as well as energy. The state space of this walk is a sphere of dimension  $3N - 4$ . The Kac conjecture concerns the spectral gap in the one step transition operator  $Q$  for this walk. In this paper, we compute the exact spectral gap.

As in previous work by Carlen, Carvalho and Loss where a lower bound on the spectral gap was proved, we use a method that relates the spectral properties of  $Q$  to the spectral properties of a simpler operator  $P$ , which is simply an average of certain non commuting projections. The new feature is that we show how to use a knowledge of certain eigenfunctions and eigenvalues of  $P$  to determine spectral properties of  $Q$ , instead of simply using the spectral gap for  $P$  to control the spectral gap for  $Q$ , inductively in  $N$ , as in previous work. We also use some deep results on Jacobi polynomials to obtain the required spectral information on  $P$ , and we show how the identity through which Jacobi polynomials enter our problem may be used to obtain new bounds on Jacobi polynomials. (Received February 26, 2007)