1026-52-4 **Elisabeth Werner***, Case Western Reserve University, Department of Mathematics, 10900 Euclid Avenue, Cleveland, OH 44106-4901,. *Convex bodies: Best and random approximation.*

How well can a convex body be approximated by a polytope?

This is a central question in the theory of convex bodies, not only because it is a natural question and interesting in itself but also because it is relevant in many applications, for instance in *computer vision, tomography, geometric algorithms, Banach spaces and stochastic geometry.*

As formulated above, the question is vague and we need to make it more precise.

Firstly, we need to clarify what we mean by "approximated". There are many metrics which can and have been considered. We will concentrate here on the symmetric difference metric which, for two convex bodies K and L in $\mathbb{R}^n$, is the volume of the symmetric difference: $d_S(K,L) = \mathrm{vol}_n(K \setminus L \cup L \setminus K)$.

Secondly, various assumptions can be made and have been made on the approximating polytopes P. For instance, one considers only polytopes contained in K or only polytopes containing K, polytopes with a fixed number of vertices, polytopes with a fixed number of $(n-1)$-dimensional faces (also called facets) etc.

Hence typical precise questions in this context are:

What is the order of magnitude of best approximation of a convex body K in $\mathbb{R}^n$ by a polytope P with a fixed number of vertices? With a fixed number of facets? We seek here the optimal dependence on all the parameters involved, like the number of vertices (or facets or k-faces), the dimension, etc.

Ideally for applications one seeks an algorithm that produces a best approximating (in a given metric) polytope. But it is only in special cases that the best approximating polytope can be explicitly singled out. Consequently, a common practice is to randomize: choose N points at random in the convex body with respect to a probability measure $\mathbb{P}$. The convex hull of these randomly chosen points is called a *random polytope*.

We then investigate the same questions for this *random approximation*. We show a remarkable result: random approximation is as good as best approximation. (Received May 10, 2006)