1026-11-8 **Jonathan Sondow*** (jsondow@alumni.princeton.edu), 209 West 97th St Apt 6F, New York, NY 10025, and **Petros Hadjicostas**. *The Generalized-Euler-Constant Function $\gamma(z)$ and a Generalization of Somos's Quadratic Recurrence Constant.*

We define the generalized-Euler-constant function $\gamma(z) = \sum_{n=1}^{\infty} z^{n-1} \left(\frac{1}{n} - \log \frac{n+1}{n}\right)$ when $|z| \leq 1$. Its values include both Euler's constant $\gamma = \gamma(1)$ and the "alternating Euler constant" $\log \frac{4}{\pi} = \gamma(-1)$. We extend Euler's two zeta-function series for $\gamma$ to polylogarithm series for $\gamma(z)$. Integrals for $\gamma(z)$ provide its analytic continuation to $\mathbb{C} - [1, \infty)$. We prove several other formulas for $\gamma(z)$, including two functional equations; one is an inversion relation between $\gamma(z)$ and $\gamma(1/z)$. We generalize Somos's quadratic recurrence constant and sequence to cubic and other degrees, give asymptotic estimates, and show relations to $\gamma(z)$ and to an infinite nested radical due to Ramanujan. We calculate $\gamma(z)$ and $\gamma'(z)$ at roots of unity; in particular, $\gamma'(-1)$ involves the Glaisher-Kinkelin constant $A$. Several related series, infinite products, and double integrals are evaluated. The methods used involve the Kinkelin-Bendersky hyperfactorial $K$ function, the Weierstrass products for the gamma and Barnes $G$ functions, and Jonquière's relation for the polylogarithm.

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