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Chaim Goodman-Strauss*, University of Arkansas, Department of Mathematics, Fayetteville, AR 72701,. *Growth, aperiodicity, and undecidability.*

Elementary questions recreational mathematics—questions that can be explained and enjoyed by children—can quickly lead to intractable or even undecidable problems. We discuss a series of such questions about tilings, their implications, and recent progress.

The wellspring for this inquiry is Hao Wang’s 1961 observation that the undecidability of the “Domino Problem” would imply the existence of an “aperiodic” set of tiles. That is, if there is no general algorithm to decide whether a given set of tiles does or does not admit a tiling of the plane, then there must exist a set of tiles that can be fitted together to form a tiling of the plane, but somehow can form only non-periodic tilings. (Since, at the time, the existence of such a set of tiles seemed preposterous, Wang incorrectly conjectured that the Domino Problem is decidable.)

Soon thereafter, the Domino Problem was proven undecidable, and aperiodic sets of tiles (most famously, the various Penrose tiles) were discovered.

Several other interesting problems have been studied since, with various implications and applications of their own, as we will discuss. (Received May 02, 2006)