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**Alexander Varchenko\*** ([anv@email.unc.edu](mailto:anv@email.unc.edu)), Department of Mathematics, UNC, Chapel Hill, NC 27599. *Multiple Orthogonal Polynomials and a Counterexample to Gaudin Bethe Ansatz Conjecture.*

It is well known that zeros of the classical Jacobi orthogonal polynomial with parameters  $\alpha, \beta$  satisfy a system of algebraic equations, which is known as the Bethe Ansatz equation of the Gaudin model associated to  $sl_2$  and two irreducible modules with highest weights  $-\alpha - 1, -\beta - 1$ . We shall discuss a generalization of this relation to the  $sl_{r+1}$  case. We consider the Bethe Ansatz equation of the Gaudin model associated to  $sl_{r+1}$  and two irreducible modules, one of which has an arbitrary highest weight and the highest weight of the other is a multiple of the first fundamental weight. This case of the Bethe Ansatz equation is a nontrivial model example to study completeness of the Bethe Ansatz method. In the considered case we solve the Bethe Ansatz equation in terms of the Jacobi-Pineiro orthogonal polynomials. As a byproduct of our relation between the Bethe Ansatz method and the theory of orthogonal polynomials, we show that the Bethe Ansatz method is incomplete for some special values of highest weights and is complete (in the considered case) for general values of highest weights. The examples of incompleteness are the first counterexamples to the Bethe Ansatz completeness conjecture for the Gaudin model. (Received August 15, 2005)